

A STUDY OF HYPERSONIC COMPRESSION-CORNER FLOW AT HIGH REYNOLDS NUMBERS

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Introduction

One of the distinguishing features of hypersonic aircrafts consists in the fact that their forebody on the windward side operates as an inlet. Here external compression of air occurs and then the air enters in the engine channel through the inlet throat. To enhance the total-pressure recovery coefficient, conditions are provided under which the flow crosses several oblique compression shocks initiated by compression corners. This flow has a complex behavior since the large increase of pressure in the shocks may cause a streamwise boundary-layer separation, which diminishes the mass-flow rate through the engine. For this reason, studies of super- and hypersonic compression-corner flows are of considerable scientific and practical interest.

The above problem was solved using either full [1-3, 5] or parabolized [3, 4, 6] Navier-Stokes equations. In the proceeding [3] numerical results of different authors are presented obtained by means of various mathematical models for laminar and turbulent flows. A comparison of the reported numerical data shows a good agreement between the predicted pressures and a considerable discrepancy between the predicted distributions of skin friction and Stanton numbers. Experimental studies of hypersonic compression-corner flows were reported in [7-9], where pressure and skin friction as well as Stanton numbers were measured both for laminar and turbulent flows.

A numerical study of a supersonic compression-corner flow, based on either full or parabolized Navier – Stokes equations for moderate Reynolds numbers ($10^5 - 10^6$), seems to be optimal. However, the ratio between the boundary-layer thickness and the reference length for a laminar flow with a high Reynolds number ($10^7 - 10^8$) is of the order of $10^{-3} - 10^{-4}$. In this case, to obtain the distributions of skin friction and Stanton numbers in the computations with a good accuracy, it is required to have a sufficiently large number of points along the normal to the body surface. For a turbulent flow, the situation at high Reynolds numbers is additionally complicated by the fact that, to accurately calculate the skin friction and Stanton numbers, one has to have a number of points (4-7) in the viscous sublayer. Since the boundary-layer thickness in universal variables is of the order of 10^3 and its relative value roughly equals 10^{-2} , the relative mesh size in the near-wall flow region should be of the order of 10^{-5} . For this reason, about 10^3 points in the normal direction is necessary to solve full Navier – Stokes numbers at high Reynolds numbers.

Besides, in solving full Navier – Stokes equations for flows with high Reynolds numbers, the error of the scheme approximation becomes in some flow regions the same order as the viscous terms [10]. For this error to be reduced, the chosen difference-scheme steps should be small enough along all spatial directions. To solve such a problem with good accuracy in the framework of full Navier – Stokes equations, one has to employ a supercomputer.

The same estimates of the necessary number of points in the normal direction are valid for the parabolized Navier – Stokes equations and the thin-layer equations. The substantial advantage of parabolized Navier – Stokes equations over the full ones is the possibility to use the marching method along the spatial coordinate. In fact, the former equations are not of

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parabolic type in the subsonic region near the body surface. It is possible to use the marching method for these equations only after applying some “regularization” in the near-wall region. It remains unclear however, how large are the errors induced to the flow parameters in this region by “regularization”, in particular, to skin friction and Stanton numbers calculated by differentiating of the velocity and temperature profiles just in the near-wall region.

For the reasons mentioned above, to numerically solve the problem for the ramp flow at high Reynolds number, it is justified to use the classical Prandtl model and to part the flow into an inviscid region and a thin boundary layer. This model is based on a rigorous asymptotic theory and describes the flow the more accurately, the higher the Reynolds number is. Since a flow separation in a compression corner may occur, the present work deals with unsteady boundary-layer equations. The approximation error in discretization of boundary-layer equations is always smaller than the value of viscid terms since the Reynolds number can be eliminated from the equations by a suitable substitution of variables. Besides, the requirement for the sufficient number of points in the cross-flow direction can be fulfilled in the case of middle-class computers since the solution is being built in a narrow near-wall region.

Statement of the problem and its solution

1. A supersonic gas flow with Mach number M_∞ around a flat plate under an angle of attack α is considered. The model has a ramp inclined at an angle β to the plate. Besides of M_∞ , α , and β , the Reynolds number Re_L based on free stream parameters and the plate length L and also the temperature factor $t_w = T_w / T_\infty$ are specified. Here the subscripts ∞ and w refer to the free stream and to the body surface, respectively.

The equations of the two-dimensional, unsteady, compressible boundary layer are written in the coordinate system (ξ, η) orientated with respect to the body surface [11-13]. The coordinates ξ and η are directed along the body surface and in the normal to it, respectively. All flow quantities are nondimensionalized by their values in the free stream and by the reference length L , whereas the pressure is normalized by $\rho_\infty w_\infty^2$, where w is the velocity-vector length. Besides, the normal velocity component v and the coordinate η are multiplied by $\sqrt{Re_L}$.

The equations is solved in the region $(\xi_1 < \xi < \xi_2, 0 < \eta < \eta_e(\xi))$. At the body surface $\eta = 0$ no-slip conditions and the condition of isothermal wall are set. At the boundary layer edge $\eta = \eta_e(\xi)$ the distribution of the longitudinal velocity $u_e(\xi)$ and temperature $T_e(\xi)$ are calculated from a given pressure $p = p(\xi)$. At the left boundary $\xi = \xi_1$ the profiles of velocity $u_1(\eta)$ and temperature $T_1(\eta)$ are determined from the Blasius solution. At the right boundary $\xi = \xi_2$, the «soft» conditions of zero second derivatives of the sought functions are posed. As the initial conditions the Blasius solution is also used.

Summary coefficients of viscosity and thermal conductivity were used to calculate the laminar, transitional, and turbulent boundary layers. These coefficients are sums of laminar terms and turbulent ones with some alternation factor Γ . For the turbulent viscosity various algebraic models of turbulence were used. The beginning and the end of the transitional region were either preset beforehand or calculated during computations depending on the value of the parameter $A = Re_\theta / \exp(0.2 M_e)$, where Re_θ is the Reynolds number based on the momentum thickness and M_e is the local Mach number at the boundary layer edge [14].

In the present analysis a two-layer implicit difference weighted scheme is employed [15]. This scheme is absolutely stable and of second order of approximation with respect to spatial variables. For the problem under consideration the scheme was modified: the ξ -directional derivatives were substituted with one-sided differences depending on the flow direction. The steady-state solution was found as an asymptotic one in time. The local skin friction c_f and Stanton numbers St , and also the displacement thickness θ are calculated from the computed velocity and temperature profiles.

$$c_f = \frac{\tau}{0,5 \rho_\infty w_\infty^2}, \quad St = \frac{q}{\rho_\infty w_\infty c_p (T_0 - T_w)}, \quad \theta = \int_0^\delta \left(1 - \frac{\rho u}{\rho_e u_e} \right) d\eta,$$

where τ and q are the friction stress and heat-flux density, respectively, and T_0 is the stagnation temperature.

The above-described algorithm was tested by two test-cases previously proposed at the International Workshop on Hypersonic Flows for Reentry Problems [3]. The first computation was performed for test-case III-2 with the following values of determining parameters $M_\infty = 10$, $\alpha = 0$, $\beta = 20^\circ$, $t_w = 5.58$ and $Re_L = 0.18 \cdot 10^5$. The obtained skin friction distribution was found to well coincide with the numerical data reported by S. Menne, W. Haase, and R. Radespiel [3]. The second computation was carried out for test-case III-4 with $M_\infty = 11.68$, $\alpha = 0$, $\beta = 15^\circ$, $t_w = 4.6$ and $Re_L = 0.25 \cdot 10^6$. The predicted skin frictions were found to well agree with the reported values everywhere except for the region $1.3 < x < 1.6$.

2. The flow in the inviscid region of a compression corner was described by the unsteady Euler equations. The effective body contour was defined as the geometric one with added displacement thickness θ . To transform the region considered into the rectangle ($0 \leq \xi \leq X$, $0 \leq \eta \leq 1$) a coordinate system (ξ, η) fitted to the effective contour was introduced. In this coordinate system the Euler equations were written in divergent form.

As the boundary conditions the zero flux were set at the surface of the effective contour $\eta = 0$, whereas the free stream conditions were used at the upper boundary $\eta = 1$ and at the entry one $\xi = 0$. The „soft“ conditions of zero second derivative of the sought function along the variable ξ were posed at the right boundary $\xi = \xi_k$.

To solve the Euler equations, one of the TVD-schemes was used, in which the derivatives along each spatial variable were approximated on a five-point pattern [16]. For time integration the five-step Runge – Kutta method was used. The scheme has the second order of approximation along all variables. The steady-state solution was found as an asymptotic one in time.

For the validation of the algorithm the computations of two test variants $M_\infty = 4$, $\alpha = 10^\circ$, $\beta = 0$ and $M_\infty = 4$, $\alpha = 0$, $\beta = 10^\circ$ were performed without a displacement thickness. The obtained pressure distributions proved to well coincide with the analytical solution.

3. Next, the whole algorithm for solving the problem of the compression-corner flow was validated. First, the solution of the Euler equations for zero displacement thickness was sought. The obtained distribution of pressure over the body surface was used in solving of the boundary-layer equations. The contour of the compression corner was corrected with the obtained displacement thickness and then the inviscid flow was computed once again.

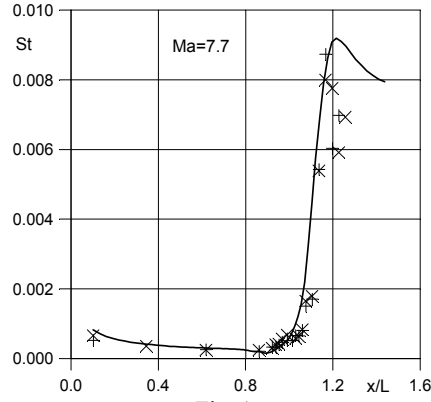


Fig. 1

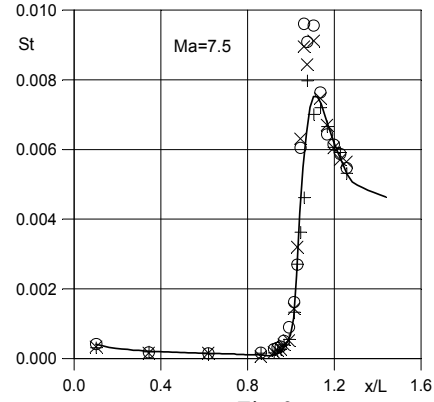


Fig. 2

Afterwards the boundary layer was recalculated with a new pressure distribution. The described iteration process was repeated until a complete convergence had been achieved.

The numerical data obtained with the described algorithm were compared with two set of experimental data [9] for the following values of determining parameters:

1) $M_\infty = 7.7$, $\alpha = 0$, $\beta = 24^\circ$, $Re_L = 1.37 \cdot 10^6$, $T_0 = 1500K$, $T_w = 300K$; and 2) $M_\infty = 7.5$,

$\alpha = 0$, $\beta = 24^\circ$, $Re_L = 5.43 \cdot 10^6$, $T_0 = 2500K$, $T_w = 300K$. The solid lines in Figs. 1 and 2

shows the distributions of the Stanton number for the two variants, while the symbols show the experimental data obtained in different measurement series. A comparison showed the agreement with the experimental data to be quite satisfactory except for several points. Thus, the applicability of the proposed algorithm was justified.

Parametric investigation

At first the effect of the surface temperature was studied for the parameters $M_\infty = 7.7$, $\alpha = 0$, $\beta = 15^\circ$, $Re_L = 5.43 \cdot 10^6$, $T_0 = 1500K$ and for the position of the transition zone $x/L = 0.2 - 0.4$. The solid lines in Fig. 3 show the predicted skin friction for the wall temperatures $T_w = 300, 800$ and $1500K$. For comparison the result for the everywhere laminar

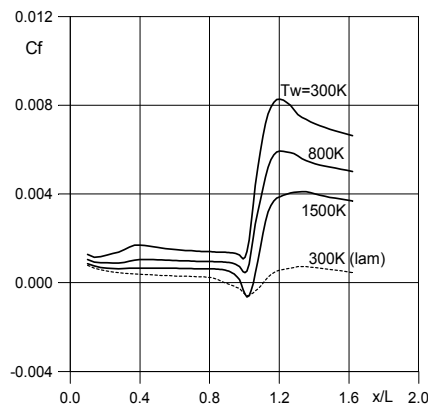


Fig. 3

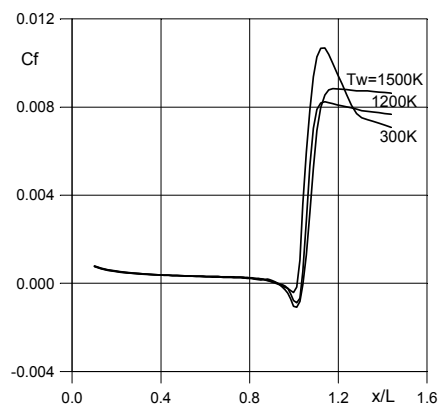


Fig. 4

flow at $T_w = 300K$ (dashed line) is shown in the same figure. It is seen that the values of the skin friction at the ramp is one order of magnitude higher in the turbulent flow than those in the laminar flow, and these values decrease as the surface temperature rises. The flow separation at the corner arises either in the laminar flow or in the turbulent one at $T_w = 1500K$ only.

The effect of the wall temperature was also examined under the assumption that the laminar-turbulent transition was in the zone $x/L = 0.99 - 1.05$ as in the comparison with experiments. In this case, the following values of determining parameters were adopted: $M_\infty = 7.5$, $\alpha = 0$, $\beta = 24^\circ$, $Re_L = 5.43 \cdot 10^6$, $T_0 = 2500K$ and the wall temperature assumed the values $T_w = 300, 1200$ and $1500K$. Figure 4 shows the skin friction distribution for the considered variants. It is seen that the flow separation length increases with increasing of the wall temperature, and the dependence of the parameter c_f on the wall temperature exhibits a non-monotonic behavior in the developed turbulent region $x/L \geq 1.1$. Thus, the flow with the transition in the separation zone substantially differs from the everywhere turbulent flow.

The dependence of the skin friction on the Reynolds number for zero angle of attack is shown in Fig. 5, where the digits near the curves refer to the following variants:

- 1 – $M_\infty = 7.5$, $\beta = 24^\circ$, $Re_{\infty,L} = 5.43 \cdot 10^6$, $T_0 = 2500K$, $T_w = 300K$, $(x/L)_{UG} = 0.99 - 1.05$
- 2 – $M_\infty = 7.5$, $\beta = 24^\circ$, $Re_{\infty,L} = 10^7$, $T_0 = 2500K$, $T_w = 300K$, $(x/L)_{UG} = 0.99 - 1.05$
- 3 – $M_\infty = 7.5$, $\beta = 24^\circ$, $Re_{\infty,L} = 10^7$, $T_0 = 2500K$, $T_w = 300K$, $(x/L)_{UG} = 0.2 - 0.4$

The values of the parameter $(x/L)_{UG}$ indicate the given intervals of the laminar-turbulent transition. As it might be expected, an increase in the Reynolds number leads to decrease of the skin friction in the turbulent region. The flow turbulization at the beginning of the plate results in an immediate growth of the parameter c_f (curve 3); however, for $x/L > 1.2$ the curve comes to the same values as in the case of turbulization in the separation zone (curve 2).

Figure 6 shows the skin friction distributions for various Mach numbers at $\alpha = 0$, $\beta = 24^\circ$, $Re_L = 5.43 \cdot 10^6$, temperature factor $t_w = 1.15$ and for the transition zone $x/L = 0.99 - 1.05$. It is seen that the separation length is slightly diminished only at $M_\infty = 10$. The skin frictions are practically coincident in the laminar region while they depend on the Mach number non-monotonically in the turbulent flow region.

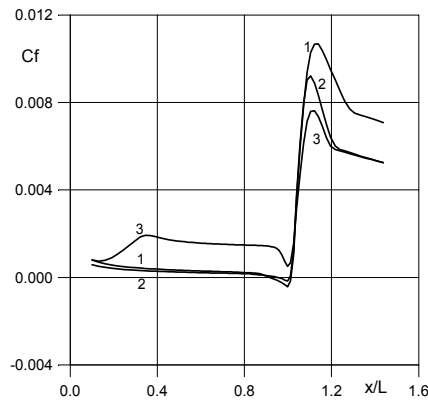


Fig. 5

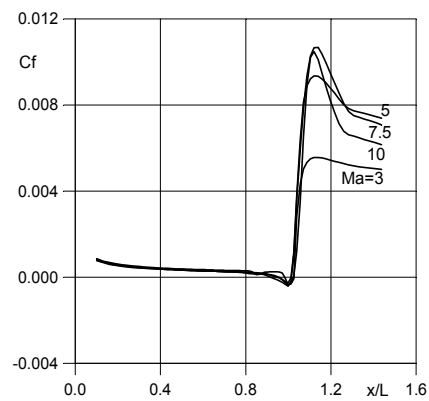


Fig. 6

Summary

The employment of the classical Prandtl model for partition of the whole flow region into an inviscid flow and a thin boundary layer is justified for a supersonic compression-corner flow at high Reynolds numbers. Computation algorithms are reported for the both regions with the permission of a weak separation as well as algorithm for solving of the problem as a whole. The proposed algorithm is validated by the comparison with the numerical results and with the experimental data. In the paper the influence of surface temperature, position of the transition region, Reynolds and Mach numbers on the skin friction has been investigated.

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